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Hence

$$b - 2AN\cos\alpha - CM + AP$$

$$= \left[1 - \frac{2\sin\gamma\cos\alpha}{\sin(\alpha+\gamma)}\right]b + \frac{\sin\beta}{\sin(\alpha+\beta)}c - \frac{\sin\beta}{\sin(\beta+\gamma)}a$$

$$= \frac{\sin(\alpha-\gamma)}{\sin(\alpha+\gamma)}b + \frac{\sin\beta}{\sin(\alpha+\beta)}c - \frac{\sin\beta}{\sin(\beta+\gamma)}a$$

$$= \left[\frac{\sin(\alpha-\gamma)\sin 3\beta}{\sin(\alpha+\gamma)\sin 3\alpha} + \frac{\sin\beta\sin 3\gamma}{\sin(\alpha+\beta)\sin 3\alpha} - \frac{\sin\beta}{\sin(\beta+\gamma)}\right]a$$

$$= \left[\frac{\sin(\alpha-\gamma)\sin 3\beta}{\sin(\alpha+\gamma)} + \frac{\sin\beta\sin 3\gamma}{\sin(\alpha+\beta)} - \frac{\sin\beta\sin 3\alpha}{\sin(\beta+\gamma)}\right]\frac{a}{\sin 3\alpha}$$

$$= \{\sin(\alpha-\gamma)[3 - 4\sin^2(\alpha+\gamma)] + \sin\beta[3 - 4\sin^2(\alpha+\beta)] - \sin\beta[3 - 4\sin^2(\beta+\gamma)]\}\, a/\sin 3\alpha$$

$$= \{\sin(\alpha-\gamma)[3 - 4\sin^2(\alpha+\gamma)] - 4\sin\beta[\sin^2(\alpha+\beta) - \sin^2(\beta+\gamma)]\}\, a/\sin 3\alpha$$

$$= \{\sin(\alpha-\gamma)[3 - 4\sin^2(\alpha+\gamma)] - 2\sin\beta[\cos 2(\beta+\gamma) - \cos 2(\alpha+\beta)]\}\, a/\sin 3\alpha$$

$$= \{\sin(\alpha-\gamma)[3 - 4\sin^2(\alpha+\gamma)] - 4\sin\beta[\sin(\alpha-\gamma)\sin(\alpha+2\beta+\gamma)]\}\, a/\sin 3\alpha$$

$$= \sin(\alpha-\gamma)[3 - 4\sin^2(\alpha+\gamma) - 4\sin\beta\sin(\alpha+2\beta+\gamma)]\, a/\sin 3\alpha$$

$$= \sin(\alpha-\gamma)[1 + 2\cos 2(\alpha+\gamma) + 2\cos(\alpha+3\beta+\gamma) - 2\cos(\alpha+\beta+\gamma)]\, a/\sin 3\alpha.$$

Now

$$\cos(\alpha+3\beta+\gamma) = -\cos 2(\alpha+\gamma)$$

and

$$\cos(\alpha+\beta+\gamma) = \cos 60^\circ = \tfrac{1}{2}.$$

Hence $b - 2AN\cos\alpha - CM + AP = 0$, and $MN = PN$.

Likewise $MP = PN$.

Also solved by T. M. Blakeslee.

CALCULUS.

335. Proposed by W. R. LEBOLD, Cambridge, Ohio.

Let $\rho = F(\theta, \phi)$ be the equation in polar coordinates of a closed surface. Show that the volume of the solid bounded by the surface is equal to the double integral

$$\frac{1}{3}\iint \rho\cos\gamma\, d\sigma$$

extended over the whole surface, where $d\sigma$ represents the element of area, and γ the angle which the radius vector makes with the external normal. [Goursat-Hedrick, *Analysis*, p. 325, ex. 9.]

SOLUTION BY S. W. REAVES, University of Oklahoma.

We may take for element of volume dV the cone (or pyramid) having the pole for vertex and the element of area $d\sigma$ for base. The altitude of this cone is clearly $\rho \cos \gamma$, and hence its volume is $dV = \frac{1}{3}\rho \cos \gamma d\sigma$. Hence the volume V, which is the sum of all the elements of volume, is, by the theory of definite integrals,

$$V = \tfrac{1}{3}\int\!\!\int \rho \cos \gamma d\sigma,$$

where the integration is to be taken over the entire surface.

344. Proposed by B. F. FINKEL, Drury College.

Solve the differential equation,

$$\frac{d^2y}{dx^2}(a^2 - x^2 - y^2) = \left(x\frac{dy}{ds} - y\frac{dx}{ds}\right)\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}.$$

SOLUTION BY A. M. HARDING, University of Arkansas.

The equation may be written in the form

$$a^2 - x^2 - y^2 = \left(x\frac{dy}{ds} - y\frac{dx}{ds}\right)\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \left(x\frac{dy}{ds} - y\frac{dx}{ds}\right)\rho,$$

where ρ is the radius of curvature at any point of the curve whose equation is sought.

Change to polar coördinates. Let ϕ be the angle the tangent makes with the initial line, and ψ the angle between the radius vector and the tangent, *i. e.*, $\phi = \theta + \psi$. Then

$$\frac{dr}{d\theta} = r \cot \psi$$

and

$$\rho = \frac{\frac{ds}{d\theta}}{\frac{d\phi}{d\theta}} = \frac{\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}}{1 + \frac{d\psi}{d\theta}} = \frac{\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}}{1 + \frac{d\psi}{dr}\cdot\frac{dr}{d\theta}} = \frac{r \csc \psi}{1 + r \cot \psi \frac{d\psi}{dr}}.$$

Also

$$\frac{dy}{ds} = \sin \phi, \qquad \text{and} \qquad \frac{dx}{ds} = \cos \phi.$$